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Detecting High Frequency Weak Signal Using Parameter-Adjusted Frequency Shifting Based on Stochastic Resonance (SR)

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Abstract

We study the occurrence of stochastic resonance phenomenon in the nonlinear bistable system when the forcing signal is at high frequency, based on the frequency shifting characteristic of Fourier transform. The simulation results show that after shifting the input frequency to a small one, and the output power spectrum peaks of the bistable system are strongly enhanced at this frequency by adjusting parameters. The ability of detecting weak signal overwhelmed in strong noise within a rather broad frequency range is improved in this paper.

Keywords- stochastic resonance; frequency shifting; bistable system; power spectrum; parameters adjustment

1. Introduction

The SR phenomenon can be observed at the output signal with small parameters, according to adiabatic approximation theory. Small parameters mean that the amplitude and frequency of period signal and noise intensity are all far less than 1[1]. SR is a nonlinear effect that accounts for the optimum response of a dynamical system to an external forcing at a precise value of the noise intensity [2-4]. The output signal-to-noise ratio (SNR) increases as the input noise intensity is increasing over a certain range [5-6]. The essence of the improvement of SNR is that the energy of noise is transformed into the signal energy. And the power spectrum of white noise changes in the structure after passing through the nonlinear bistable system [7]. It is not the uniform distribution of energy any more, but its energy focuses on the low frequency region in the form of Lorentzian[8]. Therefore there is not enough energy for the Brown

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particle to go across the double-well potential at the high frequency region, and the SR phenomenon is not be observed when the frequency of the weak signal is high [9-10].

In order to discussing this problem, we take wide parameter input signal into account. This paper presents parameter-adjusted frequency shifting stochastic resonance. First, we shift the frequency of the input signal to a small one using the frequency shifting characteristic of Fourier transform [11]. And then we take advantage of the nonlinear cooperative effect of the bistable system to get the small frequency weak signal by adjusting parameters. Finally, we shift the signal that we have detected to the frequency where it used to be.

2. Stochastic Resonance In Bistable System

Instead of reducing noise, SR system has special way of transmitting noise power into signal power to inhibit noise and strengthen useful signal. And then, we can get better detection performance than linear filter in low SNR case, and reach the goal of detecting weak signal. It is a new signal processing technology which is developing recently.

The SR system we consider in this paper can be described by the Langevin function:

$$\frac{dx}{dt} = ax - bx^3 + s(t) + n(t) \qquad (1)$$

$$\text{Then,} E[n(t)] = 0, \quad E[n(t)n(t\text{-}\tau)] = \sigma^2\delta(\tau)$$

In Fig. 1, $S_n(t) = s(t) + n(t)$ is the input signal of bistable system, $s(t) = A\sin(2\pi f_0 t)$ is the weak signal which should be detected with frequency f_0, n(t) is white noise with mean 0 and variance σ^2, and X(t) is the output of bistable system. By the effect of signal s(t) and noise n(t), output response of nonlinear bistable system X(t) has SR effect. Then, part of the noise power transmits into useful signal power, so output SNR is higher.

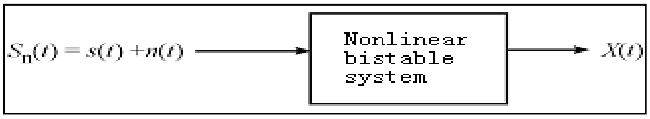


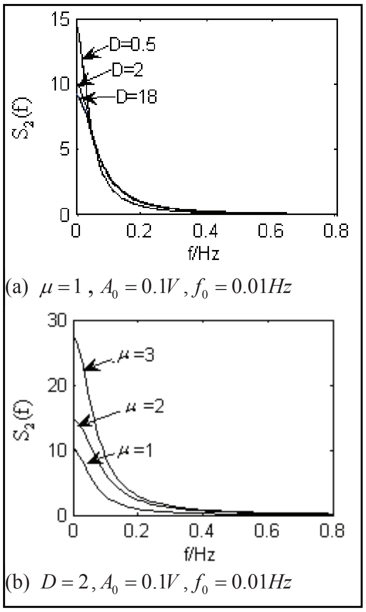
Figure 1. SR bistable system

According to adiabatic approximation theory, the output power spectrum contains two parts: One is $S_1(f)$ caused by the periodic signal and the other is $S_2(f)$ caused by noise. $S_2(f)$ shows that noise energy is concentrated in low frequency area in terms of Lorentzian distribution.

$$S_1(f) = \frac{2\mu^4 A_0{}^2 \exp(-\mu^2/2D)/(\pi D^2)}{(2\mu^2 \exp(-\mu^2/2D)/\pi^2) + (2\pi f_0)^2}\delta(f_0 - f)$$

$$S_2(f) = [1 - \frac{\mu^3 A_0{}^2 \exp(-\mu^2/2D)/(\pi^2 D^2)}{(2\mu^2 \exp(-\mu^2/2D)/\pi^2) + (2\pi f_0)^2}] \times \frac{4\sqrt{2}\mu^2 \exp(-\mu^2/4D)/\pi}{(2\mu^2 \exp(-\mu^2/2D)/\pi^2) + (2\pi f)^2)}$$

In Fig.2(a) we plot the output power spectrum $S_2(f)$ for $\mu = 1$, $A_0 = 0.1V$, $f_0 = 0.01Hz$ and some of the different noise intensity D . The output power spectrum $S_2(f)$ caused by noise decreases rapidly with the increase of frequency. The increasing of the noise intensity D has relatively little impact on the trend of $S_2(f)$ .For $D = 2$, $A_0 = 0.1V$, $f_0 = 0.01Hz$ and some of the different μ, $S_2(f)$ gets notable increase in low frequency but it is still within the low frequency limit as can be seen in Fig.2(b). So when the frequency is higher, there is no obvious effect on the generation of SR by adjusting noise intensity and parameter μ .



(a) $\mu = 1$, $A_0 = 0.1V$, $f_0 = 0.01Hz$

(b) $D = 2$, $A_0 = 0.1V$, $f_0 = 0.01Hz$

Figure 2. The trend of power spectrum $S_2(f)$ caused by noise as a function of signal frequency

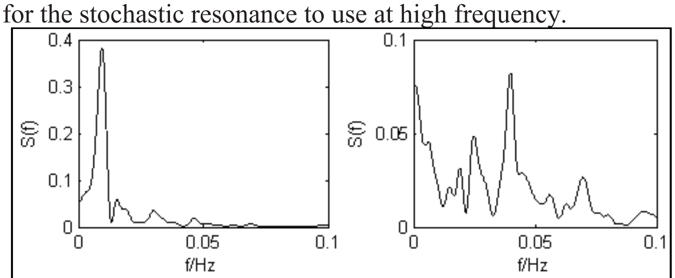
For analyzing the output frequency characteristic of the system described in equation (1), we take advantage of the Power spectrum estimation orders in MATLAB. In this paper we use the average improvement period gram law, there is the realizing function in the signal processing toolbox.

[S(f),f]= pwelch(xn,nfft,fs,window,noverlap)

Where xn is the sample sequence, S(f) is the power spectrum, nfft is the length of Fourier transform, fs is the sample frequency, window is the window function, noverlap is the overlap number of sample.

For getting a real power spectrum estimation, we should have the result of pwelch function multiplied by $\mathrm{norm(w)}^2/\mathrm{sum(w)}^2$ in which W is the window function. In this way, the power spectrum estimation will have nothing to do with the length and shape of the window function.

In order to observe the different output power spectrums at different frequencies of input periodic signals, we take $\mu = 1$, $A_0 = 0.1\mathrm{v}$, sample frequency $f_s = 5\mathrm{Hz}$ and two different small frequencies $f_1 = 0.01\mathrm{Hz}$ and $f_2 = 0.04\mathrm{Hz}$ into consideration. The power spectrum peaks at the signal frequencies can be maximized by choosing a proper noise intensity. The results are shown in Fig.3 (a) and (b). One can observe that the power spectrum peaks become small gradually with the increasing of the signal frequency. It means that the phenomenon of SR becomes weak when the frequency of the signal becomes high. There is not enough noise energy for the stochastic resonance to use at high frequency.



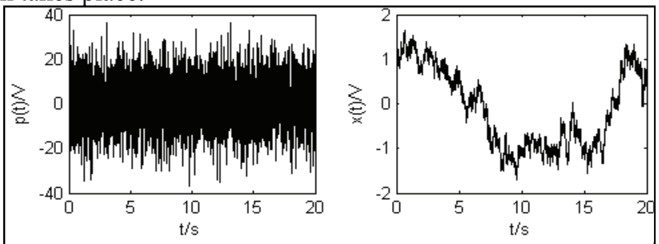
(a) f_1=0.01Hz,D=0.6962 (b)f_2=0.04Hz,D=1.3613

Figure 3. The output power spectrums with different input frequencies and noise intensities when μ =1,A0 =0.1v, sample frequency fs =5 Hz.

3. Frequency shifting stochastic resonance

In real cases, the signal parameters we encounter are often beyond the limit of small parameters, such as small frequency, noise, etc. We will focus our attention on the condition that the frequency of signal is far more than 1.As noted above, it's impossible to observe SR phenomenon when the frequency of the periodic signal is high.

Let μ =1,A_0 =0.1v, f_0=60Hz, D=60.5, fs=200Hz, the numerical analysis results of Langevin equation(1) is shown in Fig.4. We can't see a periodic output waveform which will happen if a stochastic resonance phenomenon takes place.



(a)Input signal waveform (b) Output signal waveform

Figure 4. Input and output signal waveforms when the parameters in equation (1) are μ =1,,A0 =0.1V, f0=60Hz, D=60.5, fs=200Hz.

For the sake of detecting high frequency signals, frequency shifting stochastic resonance is presented. First, $s_c(t)= \cos(2\pi f_l t)$ is multiplied by the input high frequency signal to shift its frequency to a small one. And then we detect this small frequency signal using the stochastic resonance system.

Frequency shifting:

$$S_m(t)=A_0\cos(2\pi f_0 t)\cos(2\pi f_l t)$$
$$=0.5A_0\cos[2\pi(f_0-f_l)t]+0.5A_0\cos[2\pi(f_0+f_l)t] \qquad = S_{m_1}(t)+S_{m_2}(t) \qquad (2)$$

It brings two signals which have different frequencies, $S_{m_1}(t)$ with a frequency $f_{d1}=f_0-f_l$ and $S_{m_2}(t)$ with a frequency $f_{d2}=f_0+f_l$. When frequency f_l is very close to frequency f_0, the frequency f_{d1} is very small but f_{d2} is too high to be detected.

$L_1S_{m_1}(t)+L_2S_{m_2}(t)$ is used to describe the output of the bistable system fed with $S_m(t)$. L_1 is decided by parameter μ ,noise intensity D and the frequency f_{d1} of $S_{m_1}(t)$, L_2 is decided by parameter μ ,noise intensity D and the frequency f_{d2} of $S_{m_2}(t)$ .The inequality $L_1>>L_2$ comes to existence, it can result in an increase in the signal amplitude. Therefore, the output will be a small frequency periodic signal $L_1S_{m_1}(t)$ in theory.

A simulation system is established to describe equation (1) using Simulink[14], which is shown in Fig.4. With the same parameters as in fig.3 and $f_l=59.99$ Hz, the frequency of the signal can be shifted to a small frequency 0.01 Hz and a high frequency 119.99Hz after frequency shifting. And then put this signal through the bistable system, we can see the results as shown in Fig.5. Sine Wave module becomes signal sources s(t). Random Number module which can be adjusted by Gain2 simulates white noise n(t), and is added to signals through Sum modules. Sine Wave1 provide f_l in function (2) to shift f_0 to f_0-f_l. Product module is a multiplier, which output x^3 . Gain simulates parameter a and its output is ax. Gain1 simulates parameter b and its output is bx^3 . Add module mixes input signal, outputs of modules a and b, and get the function (1). Then, the result goes through the Integrator module, which output the signal x. Using Scope module, we can observe the output signal waveform. On the other way, we can use To Workspace module to put the output data into workspace to treat it further and show its power spectrum in matlab with longa-kutta method. Or we can use Spectrum Scope module to observe the output frequency-domain waveform directly.

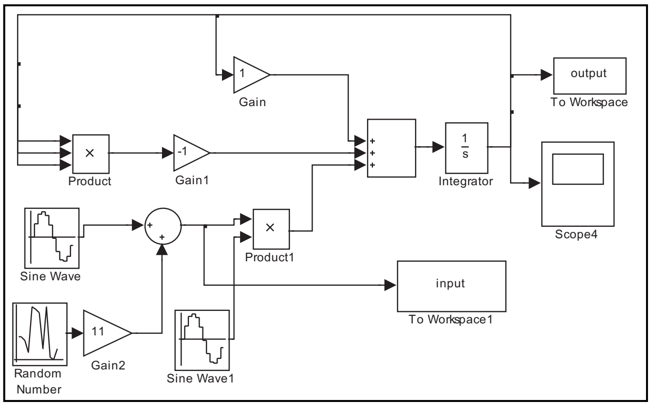
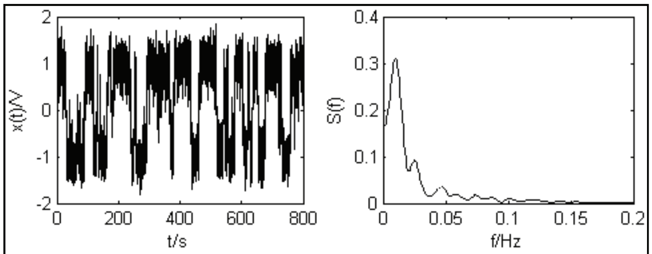


Figure 5. A simulation system for SR built by Simulink

In Fig.6(a), it shows the time-domain waveform of the output, and it's periodic. In Fig.6(b), we can observe a power spectrum peak at frequency 0.01 Hz. An effective enhancement in the power spectrum peak is observed at the small frequency 0.01 Hz. But we can't see a power spectrum peak at the high frequency 119.99Hz. This phenomenon is in accordance with the characteristic of SR. The original signal can be got from this output.



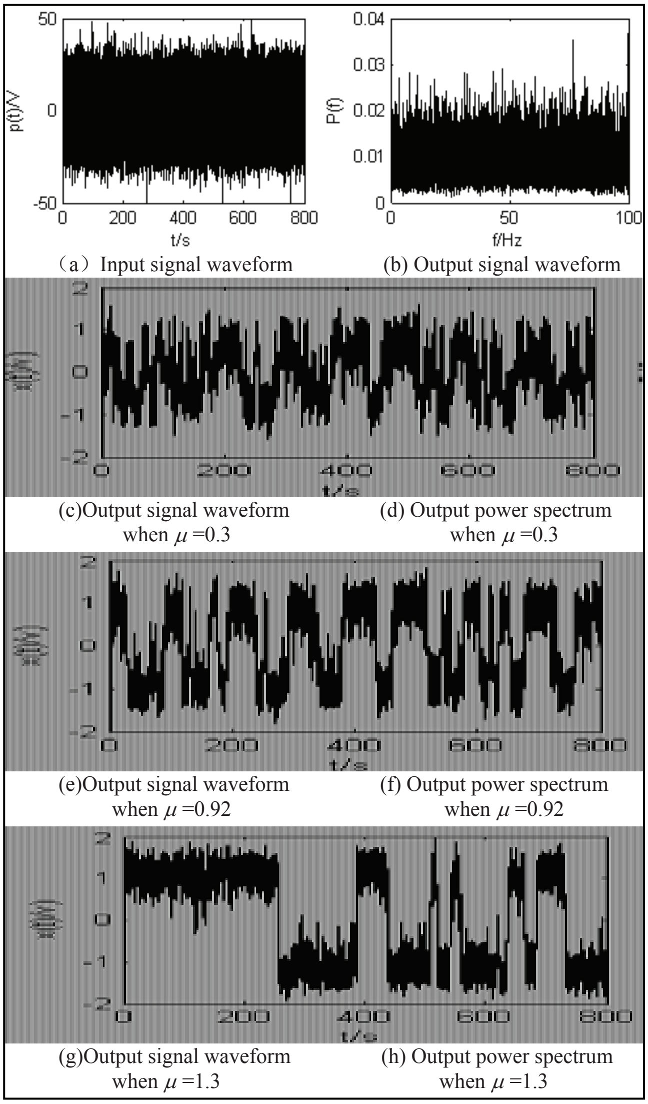
(a) Output signal waveform (b) Power spectrum of the output

Figure 6. Output signal waveform and power spectrum of frequency shifting stochastic resonance with parameters: μ =1, A0 =0.1V，f0=60Hz， fl=59.99Hz，D=60.5，fs=200Hz

4. Parameter-adjusted Frequency Shifting Stochastic Resonance

The periodic signal and noise usually mix together and we don't know the strength. When D=0, the bistable critical value of the system in equation (1) is $\sqrt{4\mu^3/27}$, which is a threshold the energy must come across to begin resonance. When statistic value A_0 gets to threshold $\sqrt{4\mu^3/27}$, the system has only one potential well left, the output will cross the barrier into another trap and a steep jump will happen. If the output can't change in the signal frequency in the common role of signal and noise, we can observe SR phenomenon by adjusting system parameter μ . The bistable critical value increases with parameter μ non-linearly.

$x=\sqrt{\mu}$ and $x=-\sqrt{\mu}$ at the signal frequency (in Fig.7 (g)).



（a）Input signal waveform (b) Output signal waveform

(c)Output signal waveform when μ =0.3 (d) Output power spectrum when μ =0.3

(e)Output signal waveform when μ =0.92 (f) Output power spectrum when μ =0.92

(g)Output signal waveform when μ =1.3 (h) Output power spectrum when μ =1.3

Figure 7. Output signal waveforms and power spectrums of frequency shifting stochastic resonance with different μ and the same parameters A0 =0.1V，f0=60Hz， fl=59.99Hz，D=60.5，fs=200Hz

The idea for detecting big frequency signal is as follows: First, shift the signal frequency to a small frequency and a big frequency. Then adjust the parameter μ gradually to change the barrier and bistable critical value for meeting the condition of SR.

Let A_0 =0.1V，f_0=60Hz，D=60.5 , fs=200Hz, the small frequency after frequency shifting is 0.01Hz. We can get a series of output waveforms and power spectrums of the stochastic resonance system with different parameter μ .

With the parameters above, we can plot the input waveform and power spectrum P(f) (in Fig.7 (a) (b)). We can't see a period signal from these two graphics.

With the varying of the system parameter μ , the barrier and the bistable critical value change correspondingly. When μ=0.3, the barrier and the bistable critical value are small. The energy of periodic signal and noise can overcome the barrier and the output state changes between $x=\sqrt{\mu}$ and $x=-\sqrt{\mu}$. But this change is not obvious in the frequency of the signal because the barrier height is too low. So there is no obvious periodic signal in the output (in Fig.7(c)), the corresponding power spectrum peak at the signal frequency is small (in Fig.7 (d)). When μ=0.92, the barrier and the bistable critical value become bigger. The energy of periodic signal and noise can just overcome the barrier and the output state changes between $x=\sqrt{\mu}$ and $x=-\sqrt{\mu}$ almost at the signal frequency (in Fig.7 (e)). The noise is suppressed effectively at the output, so SNR of the output is improved greatly. A bigger power spectrum peak emerges at the signal frequency (in Fig.7 (f)). When μ=1.3, the barrier and the bistable critical value become too big. The energy of periodic signal and noise can't overcome the barrier. The output state can't change between

5. Conclusions

In this paper, we study the phenomenon of frequency shifting stochastic resonance, and discuss the use of this phenomenon for the detection of high frequency weak signals overwhelmed in noise. From the simulation results, we can see stochastic resonance phenomenon of high frequency signal after frequency shifting. The output signal is periodic. And power spectrum peak appears just at the small frequency which we get through frequency shifting. For the mixed signal whose strength is unknown, we can detect the periodic signal using SR by adjusting the system parameters. Parameters adjustment can improve the adaptability of the system. From this we see that the adaptive of parameters is an important direction in future research. Applying at low and higher frequency signal processing, the point of this paper has bright prospect.

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